Internal Rate of Return *

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The internal rate of return (IRR) is a measure that allows for comparisons to be drawn between various investments. In technical terms, the IRR is the discount rate that sets the net present value of the investment to zero.

So what does that actually mean?

The definition refers to the net present value: which is an evaluation of the total income from a given investment in terms of today's money. An investment could refer to anything- buying stocks, starting a business, getting a degree- that generates an income stream. That income stream is then comparable to those from other investments- purchasing different stocks, taking a partnership in an existing business, choosing a different subject to study- but is also comparable to simply leaving your money in the bank.

Hence merely offering a profit isn't enough- for an investment to be worthwhile, it should offer a greater amount of profit than investing with the bank (provided, of course, you are happy with the risk implicit in the investment).

A key part of the net present value definition was that *'in today's money'* part. This ties the analysis of the income stream to interest obtainable from the bank and eventually motivates the idea of IRR. You may have guessed, therefore, that the simplest approach of adding up all the expected income doesn't quite cut it.

Instead, the model is refined by the introduction of discounting. This encapsulates the idea of time preference: that you'd rather have money now than later. This isn't purely a guard against inflation. Even if you have no need of the money now, someone else will, and you can therefore loan it to them and accrue some interest by the time you actually want to spend the original sum. You don't even have to arrange the loan yourself- deposit the money with your bank and (for a cut

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in interest) you save the effort and risk of default (unless you have the misfortune to see the bank itself fold).

So the forward direction is clear- money now can be turned into more money later by accruing interest. Working backwards, we should discount any future sum to find its value today. The reason is that you'd only need to put a smaller amount in the bank now to get the desired future sum. Here's some numbers to make things (hopefully) clear:

Example- compounded interest on £100
Suppose we place £100 in a bank account offering interest at a lump sum 5% each year (for slightly simpler calculation). Despite this simple model, the amount of cash grows faster than some would expect, as the interest becomes compounded:
After a year, there is 1.05×£100 = £105 in the bank.
At the end of the second year, interest is earnt on all the money, which includes the previously earnt £5. We get a balance of 1.05×£105 = £110.25.
For the third year we find ourselves with 1.05×£110.25 = £115.7625 at which point rounding is going to mess with the figures a little. But the general trend can be seen- after a years, at an interest rate i (expressed as a value between 0 and 1), a deposit d has attained a value of $((1+i)^a)d$.

Example- discounting £100
So suppose I offer you £100, but the catch is you won't get it for three years. How much am I really offering today? By knowing that amount, I can place it in the bank and (provided interest rates don't change) meet my obligation to you in three years through the compounded interest I earnt (and you didn't). Rearrangement of the formula says that to have a future sum s after a years at a rate i, I need a deposit of $\frac{s}{(1+i)^a}$. For our example, this requires $\frac{\pounds 100}{1.05^3} = \frac{\pounds 100}{1.157625} = \pounds 86.38$ish. (You can check this gives a £100 by applying the compound interest formula if you like).

So now we are equipped with a way to value future money. Intuitively it may have been clear that cash in the future isn't as good as cash now: you probably wouldn't accept £10000 from me now 200 years in the future because of another, more pressing time preference- your own mortality. But if you had supreme faith in medical science, you'd still be rather disappointed- at the 5% rate, that 10 grand from 2204 is worth about 58 pence today. Now we have a method to quantify this decline: a discounting process in line with the interest rate.

Of course, that 200-year case highlights a problem that was hinted at in the second example too- the interest rate is entirely likely to vary with time. This is why the move from discounting to net present value gives us a comparative tool rather than indication of what we'll actually earn: it's the earnings assuming interest rates stay as they are.

Armed with our discounting method we can determine the net present value of an income stream (a list of incomes over time) by summing the appropriately discounted values. For instance, an investment offering £100 a year is worth £100 for the first year, $\frac{£100}{1.05}$ for the next, $\frac{£100}{1.1025}$ for the next and so on. Of course, an investment that offers us £100 for doing nothing is a pretty good deal- there would most likely be an initial outlay before we generate some profits.

So let's compare two hypothetical investments. One has a higher payout but the underlying commodity wears out over time; the other pays less but returns the original investment at the end of the period. The income streams look like this:

Year :	0	1	2	3	4	5	*Sum*
A	−1000	260	260	260	260	260	300
B	−1000	75	75	75	75	1075	375

Which is the better deal? The nave approach, of adding up all the terms, rates the second one higher: £375 instead of £300. But with the first investment you get more money early on: with the discounting, can that compensate for a seemingly smaller total return? With a 5% rate, the discounted streams look like this:

Year :	0	1	2	3	4	5	*Sum*
A'	−1000	247.6	235.8	224.5	213.9	203.7	125.6639
B'	−1000	71.4	68.0	64.7	61.7	842.2	108.2369

Thus it can be seen that getting the large lump sum back at the end isn't as impressive as first thought: it's so heavily discounted that it can't make up for the (relative) losses at the beginning.

Inherent in all this is the assumption that the money earnt can be reinvested at the discount rate. But at what point does investing at that rate simply become the better plan? The net present value doesn't tell you what you'd actually get; the above investment A will only churn out £125 if interest rates stay locked at 5%. All we can usefully say is that A is better than B. To get an idea of how much, we look at the IRR-which indicates the discount rate that would be needed to set the NPV to zero, as such an investment is then equivalent to investing with the bank at that interest rate.

A precise determination of the IRR is hard to give in terms of a formula. Stream B above is analogous to a bank account (fixed interest payment, deposit returned at the end) and so the IRR is just the rate at which it pays out- 7.5% here. For more complicated structures such as Stream A, iterative techniques can be employed to home-in on an NPV of zero, exploiting the Intermediate Value Theorem to refine estimates of the discount rate. By playing around in a spreadsheet package it becomes apparent that for this example, a discount rate of 9% sets the present value at around £11, whilst a jump to 10% brings us into negative territory at -£14. Since the NPV is a continuous function of the discount rate (for sensible values), the internal rate of return will be around 9.5%.

In this case the higher internal rate of return corresponded to a higher net present value. This however, is not a general rule, although it works for valuations based on an upfront payment and subsequent earning on that payment. If a large payment is required near the end of the life of a project, then a better investment (in terms of net present value) may have a *lower* internal rate of return:

$Year:$	0	1	2	3	4	5	Sum
A	-150	220	220	220	220	-850	-120
B	-150	240	240	240	240	-850	-40

Here stream B is strictly better than stream A- they have the same outgoings, at the same time, but in the profit years 1-4, B performs better. Yet, the internal rate of return for A is somewhere around 7.5%, higher than that of B which stands at around 2.5%. Indeed, at precisely 7.5% stream A is returning a loss of £5 whilst stream B is turning a profit of nearly £62.

Care then must be exercised in the application of the internal rate of return to the valuation of an investment, but it nonetheless remains a major tool of financial analysis. In particular, it plays a key role in the valuation of Bonds, where it is known as the Yield to Maturity. Whilst the coupon (interest rate paid by the Bond) and face value are fixed, in the market the purchase price will vary with the yield due to fluctuations in the risk-free interest rate. The sensitivity of the price to these changes has been modelled by increasingly sophisticated methods over the years, such as duration (introduced in the 30s), modified duration, and convexity. These valuations in turn shape the pricing and delivery of Bond futures, which are based on a notional Bond for which the cheapest to deliver of the real Bonds available in the market must be calculated.